

Indian Math Online – Solution Explanation HCF (Highest Common Factor)

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Highest Common Factor

The greatest number among all the common factors of two or more than two numbers is called the highest common factor (HCF) or greatest common divisor (GCD).

Example:

Q. Find the highest common factor of 45, 80, and 110.

Explanation: All the factors of

45 are: 1, 3, 5, 9, 15 and 45

80 are: 1, 2, 4, 5, 8, 10, 16, 20, 40 and 80.

110 are: 1, 2, 5, 11, 22, 55 and 110.

The common factors are 1 and 5.

The greatest among all the common factors is 5. So, **HCF = 5**

HCF by Prime factorization method

In this method find the prime factorization of the given numbers. Then take all the common prime factors and find their product. The product of these common prime factors is the HCF of the given numbers.

Let us solve the above example in this method.

Examples:

Q.1. Find the highest common factor (HCF) of 45, 80, and 110.

Explanation: Find prime factorization of the given numbers.

$$\begin{array}{r|l} 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 80 \\ \hline 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 110 \\ \hline 5 & 55 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

$$45 = 3 \times 3 \times 5$$

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$

$$110 = 2 \times 5 \times 11$$

The common factor of all the numbers is 5.

The highest common factor is **5**.

Q.2. Find the highest common factor (HCF) of 60, 100, and 140.

Explanation: Find the prime factorization of all the given numbers.

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$100 = 2 \times 2 \times 5 \times 5$$

$$140 = 2 \times 2 \times 5 \times 7$$

The common factors of all the numbers are 2, 2, and 5.

To find the HCF multiply all the common prime factors.

$$2 \times 2 \times 5 = 20$$

So, HCF (highest common factor) is 20.

Note: If one number is the factor of other, the smaller number is the HCF of the given numbers.

Q.3. Find the HCF (highest common factor) of 44, 132.

Explanation: Observe that $44 \times 3 = 132$

Therefore 44 is a factor of 132.

So, HCF of the numbers 44 and 132 is 44.

HCF by Short Division Method

To find HCF by short division method, divide all the numbers together by their common prime factors. Stop the division when no common prime factor is left.

Find the product of the common prime factors.

The product of these factors is the HCF of the given numbers.

Let us solve the above problem in this method.

Examples:

Q.1. Find the highest common factor (HCF) of 60, 100, and 140

Explanation: Find the common prime factors in short division method.

2	60, 100, 140
2	30, 50, 70
5	15, 25, 35
	3, 5, 7

Since 3, 5 and 7 have no common prime factors, we stop the division here.

The common factors are 2, 2, and 5.

The highest common factor is $2 \times 2 \times 5 = 20$.

Q.2. Find the HCF (highest common factor) factor of 120, 250.

Explanation: Use short division method to find HCF

2	120, 250
5	60, 125
	12, 25

12 and 25 have no common prime factors. So we stop the division here.

The HCF of 120 and 250 is $2 \times 5 = 10$.

Q.3. Find the HCF (highest common factor) of 36, 180 and 1260.

Explanation: Use short division method to find HCF

2	36, 180, 1260
2	18, 90, 630
3	9, 45, 315
3	3, 15, 105
	1, 5, 35

1, 5 and 35 have no common prime factors. So we stop the division here.

The HCF of 36, 180 and 1260 is $2 \times 2 \times 3 \times 3 = 36$.

HCF by Long division Method

When the numbers are large, the short division method is tedious. We use long division method to find HCF of large numbers. In this method the larger number is divided by the smaller number. The remainder is used to divide the divisor again and this continues till we get 0 as remainder.

The last divisor is the HCF of the given numbers.

Examples:

Q.1. Find HCF (highest common factor) of 2040 and 3600.

Explanation: We shall use long division method to find HCF as the given numbers are large.

$$\begin{array}{r}
 \begin{array}{r}
 \overline{1} \\
 2040 \overline{) 3600} \\
 \underline{2040} \quad 1 \\
 1560 \overline{) 2040} \\
 \underline{1560} \quad 3 \\
 480 \overline{) 1560} \\
 \underline{1440} \quad 4 \\
 120 \overline{) 480} \\
 \underline{480} \\
 0
 \end{array}
 \end{array}$$

The last divisor is 120 and the remainder is 0.
Now, **120** is the HCF of 2040 and 3600.

Q.2. Find the HCF of 480 and 360.

Explanation: We divide 480 by 360, and then divide 360 by the remainder and so on.

$$\begin{array}{r}
 \begin{array}{r}
 \overline{1} \\
 360 \overline{) 480} \\
 \underline{360} \quad 3 \\
 120 \overline{) 360} \\
 \underline{360} \\
 0
 \end{array}
 \end{array}$$

So, **120** is the HCF of 360 and 480.

Properties of HCF

1. The HCF of a group of numbers is not greater than any of the given numbers.
2. The HCF of two prime numbers is 1.
3. The HCF of two co-prime numbers is 1.

