

Indian Math Online – Solution Explanation Introduction to Integers

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So far we have learnt about Natural Numbers $N = \{1, 2, 3, \dots\}$ and
Whole Numbers $W = \{0, 1, 2, 3, \dots\}$

Now consider the following situations:

What happens if a larger number is to be subtracted from a smaller number?

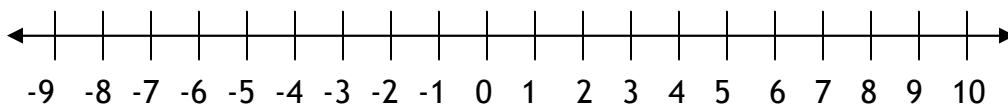
How do we represent the temperature below 0°C ?

How do we find the distance between an airplane and a submarine?

Clearly, no whole numbers can be used to represent these situations. Thus the negative numbers are introduced. They are denoted by $-1, -2, -3 \dots$ read as minus one, minus two, minus three etc.

Introduction

The **integers** are the set of numbers including the whole numbers (0, 1, 2, 3.....) and their negatives (0, -1, -2, -3...).



The number line is a line labeled with the integers in increasing order from left to right, that extends in both directions.

The arrows on each end of the number line show us that the line stretches to infinity in both the directions. We don't have to include a positive sign (+) when we write positive numbers. However, we do have to include the negative sign (-) when we write negative numbers.

Zero is called the origin, and it's neither negative nor positive.

The set of integers is represented by $Z = \{\dots -3, -2, -1, 0, 1, 2, 3, 4 \dots\}$

Two integers that are at the same distance from zero in opposite directions are called opposites.

Example: 3 and -3 are at a distance of 3 units from the origin but 3 is in right of 0 and -3 is in left of 0.

$1 + (-1) = 0$, $2 + (-2) = 0$, $3 + (-3) = 0$ and so on.

1 and (-1) are opposites of each other.

2 and (-2) are opposites of each other.

3 and (-3) are opposites of each other and so on.

Ordering of Integers

If two integers are represented on the number line, by convention we know that the number occurring on the right is greater than that on the left. The number on the left is smaller than that on the right.

Thus, (i) $5 > 2$, since 5 is to the right of 2.

(ii) $3 > 0$, since 3 is to the right of 0.

(iii) $0 > (-5)$, since (-5) is to the left of 0.

(iv) $(-2) > (-6)$, since (-6) is to the left of (-2).

(v) $9 > 5$ and $(-9) < (-5)$

These observations can be generalized as follows:

1. Zero is less than every positive integer, since 0 lies to the left of every positive integer.
2. Zero is greater than every negative integer, since 0 lies to the right of every negative integer.
3. Every positive integer is greater than every negative integer, since every positive integer lies to the right of every negative integer.
4. The greater the number, the lesser is its opposite.

Absolute Value of Integer

The absolute value of the integer is the numerical value of the integer regardless of its sign.

Example: Consider integer -4 on the number line. It is at distance of 4 units from 0 to the left side. Integer '+4' is also at a distance of 4 units from 0 to the right side. So absolute value means the distance of number from 0.

Absolute value is always denoted by the symbol '| |'.

$$|+4| = 4 \text{ and } |-4| = 4$$

The absolute value of a number is always a positive number.

The opposites are at the same distance from the origin, so they have the same absolute value.

The absolute value of zero is zero.

Examples:

$$|-2186| = 2186$$

$$|891| = 891$$

$$|0| = 0$$

$$|12| = 12$$

$$|-4| = 4$$

Negative of a negative integer

The negative of a negative integer is a positive integer.

Example: $-(-8) = 8$

$$-(-412) = 412$$

Examples:

Q.1. What integer could you use to represent the phrase "15 units right to 0 on the number line"?

Explanation: Any number to the right of 0 on the number line is a positive integer. A number which is 15 units to the right of 0 is **+15**.

Q.2. What integer could you use to represent the phrase "loss of \$500"?

Explanation: Since it is a loss, it is represented as a negative number. Therefore a loss of \$500 is represented as **-\$500**.

Q.3. Indicate the following using appropriate sign:

12° below freezing point of water

Explanation: The temperature below freezing point is represented by a negative sign. Therefore 12° below freezing point is represented by **-12°**.

Q.4. Indicate the following using appropriate sign:

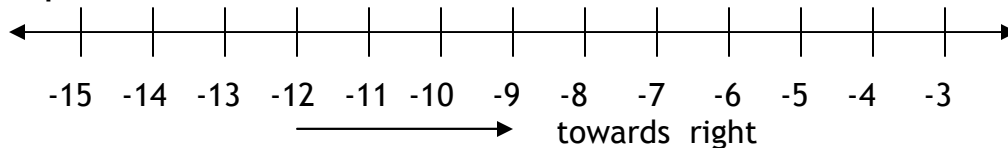
Gaining weight of 23 lb

Explanation: Gaining is represented by positive sign. Therefore gaining a weight of 23 lb is represented by **+23 lb**.

Q.5. -4 is to the right of -14 on the number line.

- True
- False

Explanation:



Observe in the above number line that -4 is to the right of -14.

The given statement is **true**.

Q.6. -1 is the smallest negative number.

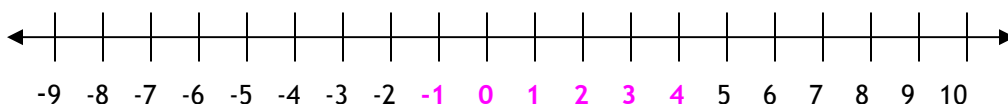
- True
- False

Explanation: As the numeral increases, the value of the negative integer decreases. So, -1 is the largest negative integer not the smallest.

The given statement is **false**.

Q.7. Find all the integers between -2 and 5.

Explanation:



See the number line above and write the integers between -2 and 5. They are **-1, 0, 1, 2, 3 and 4**.

Q.8. What is the value of $|-5|$?

Explanation: The absolute value of -5 is the numerical value of the integer with out the sign.

Therefore $|-5| = 5$

Q.9. $|-12| < |-5|$

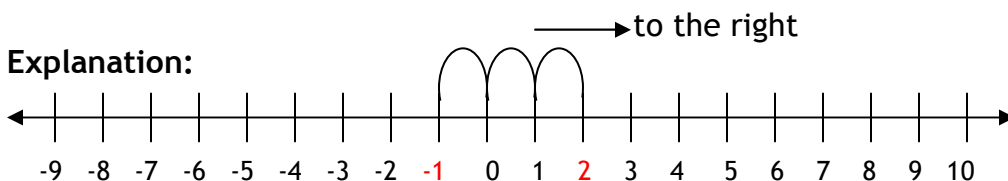
- True
- False

Explanation: $|-12| = 12$ and $|-5| = 5$

Now, $12 > 5$, therefore $|-12| > |-5|$. The above statement is **false**.

Q.10. To which number do we reach if we move 3 numbers to the right of -1 on the number line?

Explanation:



The number which is 3 numbers to the right of -1 is **2**.

Q.11. Pick the largest integer.

- 31
- 13
- 12
- 10

Explanation: The positive integers are greater than the negative integers. Therefore 12 and 10 are greater than the other two integers -31 and -13. Among 12 and 10, 12 is the greater number. Therefore, **12** is the largest among the given integers.

Q.12. Pick the smallest integer.

- 34
- 45
- 65
- 0

Explanation: All the negative integers are smaller than the positive integers. Therefore -34 and -45 are smaller than the other two integers 65 and 0. Among the negative integers i.e. -45 and -34; -45 is smaller than -34. Therefore the smallest integer is **-45**.

Q.13. Fill in the blank using appropriate symbol; $<$, $>$, $=$:
-65 _____ 0

Explanation: All the negative numbers are less than 0.
Therefore $-65 < 0$

Q.14. -12 is greater than -10.

- True
- False

Explanation: We know that $12 > 10$. As the number increases the value of its opposite decreases. Therefore, -12 is less than -10.
The given statement is **false**.

Q.15. Arrange 5, -7, -2, 8 in descending order.

Explanation: 5 and 8 are positive integers and $8 > 5$.

-7 and -2 are negative integers and $-2 > -7$.

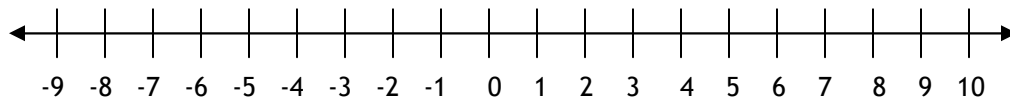
All the negative integers are smaller than the positive integers.

So, the given numbers arranged from largest to smallest are **$8 > 5 > -2 > -7$**

Q.16. On the number line which integer is at the least distance from zero?

- 1
- 2
- 3
- 4

Explanation: See the number line below



-1 is at a distance of 1 unit from 0

-2 is at a distance of 2 units from 0

-3 is at a distance of 3 units from 0 and

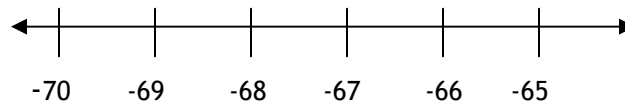
-4 is at a distance of 4 units from 0

Therefore **-1** is at the least distance from zero.

Q.17. The predecessor of -67 is -68.

- True
- False

Explanation: The predecessor of a number is the number which comes just before the given number.



Observe that (-68) is one unit to the left of (-67) on the number line.

(-68) comes just before (-67).

Therefore (-68) is the predecessor of (-67). The given statement is **true**.

Q.18. The successor of -12 is -13.

- True
- False

Explanation: The successor of a number is the number that comes just after the given number.



Observe that (-13) is one unit to the left of (-12) on the number line.

So, (-13) come just before (-12).

Therefore (-13) is the predecessor of (-12). The given statement is **false**.