

## Indian Math Online – Solution Explanation Rational Numbers And Its Properties

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We have already studied about natural numbers, whole numbers, integers and fractions. Let's recall them once.

**Natural numbers:** Numbers used for counting are called natural numbers.  
Example: 1, 2, 3.....

**Whole numbers:** All natural numbers and zero together are called whole numbers. Example: 0, 1, 2, 3.....

**Integers:** The whole numbers together with negatives of counting numbers are called integers.  
Example: -5, -4, -1, 0, 1, 3, 4.....

**Fractions:** Numbers of the form  $a/b$ , where  $a$  &  $b$  are natural numbers are called fractions.

Consider a simple equation in one variable:

$$2x - 5 = 0$$

We can verify that  $5/2$  is the solution of this equation.

Now consider another equation:

$$3x - 9 = 0$$

We can verify that 3 is the solution of the equation.

Consider one more equation:

$$2x + 5 = 0$$

We find that no integer or fraction satisfies this equation. We need to extend our number system to include numbers which can satisfy this type of equation. This new system is called rational number system.

**Rational numbers:** The number of the form  $p/q$ , where  $p$  &  $q$  are integers and  $q \neq 0$ , is called a rational number.

Example:  $4/3$ ,  $-28/65$ ,  $0/90$  etc.

The integer 'p' of the rational number ' $p/q$ ' is called the numerator and 'q' is called the denominator.

Example:  $-6/17$  has -6 as numerator and 17 as denominator.

**Points to be noted:**

1. 0 is considered as a rational number as we can write 0 as  $0/1$  which is quotient of two integers with non- zero denominator.
2. Every natural number is a rational number as they can be written in  $p/q$  form with non zero denominator.  
**Example:**  $5 = 5/1$ ,  $100 = 100/1$
3. Every integer is a rational number as they can be written in  $p/q$  form with non zero denominator.  
**Example:**  $-5 = 5/1$ ,  $-100 = 100/1$
4. Every fraction is a rational number.

**Positive rational numbers:** If both numerator and denominator are positive, the rational numbers is said to be positive.

**Example:**  $4/5$ ,  $17/21$ ,  $101/300$

If both numerator and denominator are negative, then also the rational numbers is said to be positive.

**Example:**  $\frac{-5}{-11}$ ,  $\frac{-56}{-103}$ ,  $\frac{-200}{-849}$

**Negative rational numbers:** If either numerator or denominator is negative, then the rational is said to be negative.

**Example:**  $\frac{-6}{13}$ ,  $\frac{18}{-33}$ ,  $\frac{-10}{23}$ ,  $\frac{43}{-47}$

**Equivalent rational numbers:** Two rational numbers ( $p/q$ ) and ( $m/n$ ) are said to be equivalent if,  $p \times n = m \times q$ .

**Example:**  $\frac{1}{4}$  and  $\frac{4}{16}$  are equivalent fractions as  $1 \times 16 = 4 \times 4 = 16$

Equivalent rational numbers represent the same rational number.

To find an equivalent rational number of rational number  $a/b$ , we have to either divide or multiply both the numerator and denominator a & b respectively by the same number except 0.

$\Rightarrow$  If  $\frac{a}{b}$  is rational number then  $\frac{a \times p}{b \times p}$  is an equivalent rational number to  $\frac{a}{b}$

$\Rightarrow$  If  $\frac{a}{b}$  is rational number then  $\frac{a \div p}{b \div p}$  is an equivalent rational number to  $\frac{a}{b}$

**Examples:**

**Q.1.** Is -13 a rational number?

**Explanation:** -13 can be written as  $-13/1$  which is in  $p/q$  form where  $q$  is not equal to zero. Hence -13 is a rational number.

**Q.2.** What is the numerator & denominator of the rational number  $5/-11$  ?

**Explanation:** Numerator is 5 and the denominator is -11.

**Q.3.** Are  $-15/18$  and  $5/-6$  equivalent rational numbers?

**Explanation:**  $\frac{-15}{18} \swarrow \nearrow \frac{5}{-6}$

Cross multiply the given rational numbers

$$(-15) \times (-6) = 90 \text{ and } 18 \times 5 = 90$$

$$\text{So, } (-15) \times (-6) = 18 \times 5$$

Hence, the given rational numbers are equivalent.

**Q.4.** Fill in the blank:

$$\frac{3}{5} = \frac{\dots}{25}$$

**Explanation:** By what number should we multiply 5 to get 25?

$$\text{Clearly } 5 \times 5 = 25$$

$$\text{Therefore, } \frac{3}{5} = \frac{3 \times 5}{5 \times 5} = \frac{15}{25}$$

**Q.5.** For what value of  $x$ ,  $-3/8$  and  $6/x$  are equivalent rational numbers?

**Explanation;**  $-3/8$  and  $6/x$  are equivalent rational numbers. Therefore,

$$\frac{-3}{8} \swarrow \nearrow \frac{6}{x}$$

By cross multiplying we get,  $(-3) \times x = 6 \times 8$

$$\Rightarrow -3x = 48$$

$$\Rightarrow x = 48/(-3)$$

$$\Rightarrow x = -16$$

**Q.6.** Express  $-65/39$  as rational number with numerator 5.

**Explanation:** By what number should we divide  $-65$  to get 5?

Clearly,  $-65 \div (-13) = 5$

Therefore,  $\frac{-65}{39} = \frac{-65 \div (-13)}{39 \div (-13)} = \frac{5}{-3}$

Hence,  $\frac{-65}{39} = \frac{5}{-3}$

**Note:**  $\frac{3}{-4} = \frac{-3}{4} = -\frac{3}{4}$